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A PECULIARITY IN MEASURING THE DISPERSE DROP COMPOSITION IN A TWO-PHASE STREAM

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UDC 532.529.5

The paradox of repeated drop breakup in a two-phase stream is considered. A method of measuring their slip coefficients is proposed.

The criterial condition for stability of drops [1]

$$\frac{2r\rho_g u^2}{\sigma} \leqslant \frac{10.6}{\sqrt{1-\frac{r_{min}}{r}}} \tag{1}$$

is known in computations of the heat and mass transfer associated with air atomization of fluids.

The existence of a limit drop size $r \leq r_{min}$ which is stable in any high-speed stream dictated the mutual influence of aerodynamic forces and inertia and deformability of the drops [1]. For water $r_{min} = 23\ \mu$. At the same time, it has been detected in [2] that the maxima of the mass spectra of water drops determined by the method of light scattering in the cylindrical part of a Venturi tube (throat) shift toward smaller sizes as the checked section is removed from the entrance into the throat. The authors interpreted this fact as the result of repeated breakup of the drops of turbulent gas pulsations, which contradicts (1). Indeed, according to the test conditions in [2], the velocity of blowing the coarsest $u_0 = 120$ m/sec in the first checking section ($x_1 = 30$ mm) for $u_0 = 120$ m/sec was known to be less than 120 m/sec, and therefore [according to (1)] drops of radius $\leq 50\ \mu$ should be stable under subsequent transportation.

The continuous breakup of drops in a turbulent stream observed in [3] does not contradict (1), but also does not confirm the deduction in [2], since the immiscible fluids in [3] possessed the identical density and surface tension (fine-scale turbulent pulsations were studied), while $\rho_g \ll \rho_f$ in [2].

We tried to show the presence of another, purely kinematical, reason for the transformation of the size spectrum observed in the section of drop acceleration by the gas stream. We have in mind one peculiarity in measuring the disperseness in a two-phase stream, which has still not received a clear exposure in the literature.

As is known, the mass spectrum $g(r, x)$ in a two-phase stream is understood to be the mass fraction of drops supplied by unit volume of gas. Hence, the flow rate of drops, whose radii lie in the interval dr, in a section x is

$$dq(r, x) = Q_f g(r, x)\, dr = mg(r, x)\, v(x) S(x)\, dr, \tag{2}$$

since $m = Q_f/Q_g$ and $Q_g = v(x)S(x)$.

If the drops being transported in the interval $\{x_1 - x_2\}$ retain their size (no coagulation or breakup), then $g(r, x_1) = g(r, x_2)$.

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Besides the function g(r, x) it is possible to introduce also $g_s(r, x)$ and to write the drop flow rate in the following form:

$$dq(r, x) = Mg_s(r, x)v(r, x)S(x)dr. \tag{3}$$

Equating (2) and (3) and taking account of normalization of both distributions

$$\int_0^\infty g(r, x)dr = \int_0^\infty g_s(r, x)dr = 1 \tag{4}$$

we obtain after simple manipulation

$$g_s(r, x) = \frac{v(x)}{v(r, x)} g(r, x) \Big/ \int_0^\infty \frac{v(x)}{v(r, x)} g(r, x)\, dr, \tag{5}$$

$$g(r, x) = \frac{v(r, x)}{v(x)} g_s(r, x) \Big/ \int_0^\infty \frac{v(r, x)}{v(x)} g_s(r, x)\, dr. \tag{6}$$

All the methods of recording the drop disperseness in a two-phase stream must be separated into two classes according to (5)-(6): in the first, the true (continuous) spectrum g(r, x), and in the second, the "photographic" spectrum $g_s(r, x)$.

The first class includes the method of drop deposition on an immersion layer [4], the pulse-counting method described in [5], the continuous photoelectric counter, etc.

The light-scattering method is among the photographic class of methods. The readings of the recording device (a millivoltmeter) are coupled functionally to the concentration of drops in the exposed volume of the pipeline. As follows from (5), the size spectrum measured by this method does not retain its shape on the section of accelerated drop motion even in the absence of coagulation and breakup, were $g_s(r, x) \to g(r, x)$ as $x \to \infty$.

Transformations of the spectra $g_s(r, x)$ when going from the section $x_1 = 10$ mm to the section $x_2 = 300$ mm are shown in Fig. 1 for a typical g(r, x) distribution [5]. A change in the drop velocity in the cylindrical section was traced by means of the known equation [1]

$$\rho_f \frac{4}{3}\pi r^3 \frac{dv(r, x)}{dt} = -\frac{1}{2}\frac{14}{\sqrt{\mathrm{Re}}}\pi r^2 u^2(r, x), \tag{7}$$

whose solution with the boundary and initial conditions of interest to us [$v(x) = u_0$, $v(r, 0) = 0$] has the form

$$t = B\left[\frac{1}{\sqrt{u}} - \frac{1}{\sqrt{u_0}}\right], \tag{8}$$

$$x = \frac{B}{\sqrt{u}}\left[\sqrt{u_0} - \sqrt{u}\right]^2, \tag{9}$$

where

$$B = 4\sqrt{2}\, r^{3/2}\rho_f / 21\sqrt{\mu\rho_g}.$$

The following values are given to the parameters entering into (8) and (9): $\rho_g = 1.1$ kg/m^3, $\rho_f = 10^3$ kg/m^3, $\mu = 16 \cdot 10^{-6}$ nsec/m^2, and $u_0 = 100$ m/sec. The distribution curves are plotted to the accuracy of the normalizing constant C.

It is easy to see from the graphs presented and the relationship (5) that the velocities of both phases flatten out at a spacing of ≈300 mm from the entrance to the throat.

A kinematic transformation of the spectra $g_s(r, x)$ discloses the possibility of measuring the drop slip coefficients [v(r, x)/v(x)] by using the apparatus of the light-scattering method.

Let the spectra $g_s(r, x_1)$ and $g_s(r, x_2)$ be recorded in the section 1 and 2 of a pipeline. Here and henceforth, the subscripts 1 and 2 refer, respectively, to the check section and to where the velocities of both phases flatten out. This latter will be satisfied if $x_2 - x_1 \approx 0.5$ m. Moreover, let us consider that the drop stability condition is satisfied in the interval $\{x_1 - x_2\}$, while drop coagulation exerts no substantial influence (see below) on transformation of the spectrum g(r, x). Then for steady flow

$$M_1 g_s(r, x_1)v(r, x_1)S_1 dr = M_2 g_s(r, x_2)v(r, x_2)S_2 dr. \tag{10}$$

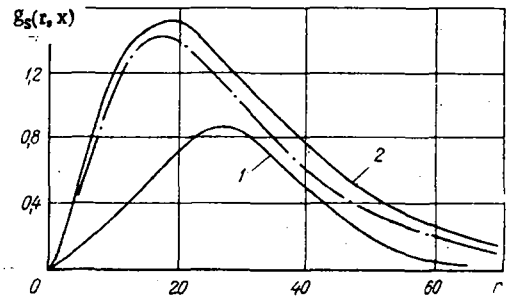

Fig. 1. Theoretical drop size distributions $g_s(r, x)$ at a spacing of 10 mm (1) and 300 mm (2) from the entrance to the throat. The dashed-dot curve is the distribution $g(r, x)$: r, μ.

Since $v(r, x_2) = v(x_2)$ and $v(x_1)S_1 = v(x_2)S_2$, (10) is rewritten as follows:

$$\frac{v(r, x_1)}{v(x_1)} = \frac{M_2}{M_1} \frac{g_s(r, x_2)}{g_s(r, x_1)}. \tag{11}$$

Using the law of attenuation of the intensity of a parallel light beam passing through a layer of polydispersed aerosol of thickness l [6]

$$I(l) = I(0) \exp\left[-Nl \int_0^\infty K(r) f_s(r) \pi r^2 dr\right], \tag{12}$$

as well as the relationship between the counting and mass concentrations

$$M = \frac{4}{3} \pi \rho_f N \int_0^\infty f_s(r) r^3 dr, \tag{13}$$

we have

$$M = \frac{4}{3} \frac{1}{\bar{K}} \rho_f \frac{\bar{r}}{l} \ln \frac{I(0)}{I(l)}, \tag{14}$$

where

$$\bar{r} = \int_0^\infty f_s(r) r^3 dr \Big/ \int_0^\infty f_s(r) r^2 dr, \tag{15}$$

$$\bar{K} = \int_0^\infty K(r) f_s(r) r^2 dr \Big/ \int_0^\infty f_s(r) r^2 dr. \tag{16}$$

If it is taken into account that $K = 2$ for particles of size $\geq 2\ \mu$ (see [10]), then we obtain from (11) and (14)

$$\frac{v(r, x_1)}{v(x_1)} = \frac{g_s(r, x_2) \bar{r}_2 l_1}{g_s(r, x_1) \bar{r}_1 l_2} \left[\frac{\ln I(0)/I(l_2)}{\ln I(0)/I(l_1)}\right]. \tag{17}$$

From the viewpoint of spatial and structural resolution, laser diffraction structure meters possessing a fine monochromatic light beam are most preferable for drop velocity measurements according to the algorithm (17).

The algorithm (17) is suitable for aerosols of any disperseness, including monodisperse, but is rather awkward in technical respects. In the typical case of a continuous spectrum, the ratio of the mass concentrations in (11) can be found by a simpler means [7] by using the passage to the limit

$$\frac{M_2}{M_1} = \lim_{r \to 0} \frac{v(r, x_1)}{v(x_1)} \lim_{r \to 0} \frac{g_s(r, x_1)}{g_s(r, x_2)}. \tag{18}$$

Since $v(r, x_1) \to v(x_1)$ as $r \to 0$, then (11) becomes

$$\frac{v(r, x_1)}{v(x_1)} = \frac{g_s(r, x_2)}{g_s(r, x_1)} \lim_{r \to 0} \frac{g_s(r, x_1)}{g_s(r, x_2)}. \tag{19}$$

The confidence in the slip coefficients computed by means of (19) is justified by the fact that the ratio $v(r, x_1)/v(x_1)$, and therefore $g_s(r, x_2)/g_s(r, x_1)$, varies between insignificant limits (10-30%) in the range $r \leq 10\,\mu$.

As has been noted above, the relationships (17) and (19) are valid when the probability of any drop from the size spectrum passing through all the rest is close to one in the section $\{x_1 - x_2\}$. In turn, this means that the effective volume being treated by the drops per unit volume of gas is considerably less than one; i.e.,

$$n\int_0^\infty \pi r^2 u(r, x)\,dt \ll 1. \tag{20}$$

According to the recommendations in [8], the polydisperse flow in (20) is replaced by a monodisperse flow with the effective radius r; $\pi\bar{r}^2 u dt$ is the volume processed by the drops in the time dt.

Using (9) and (13) in (20), we have

$$m \ll \frac{7}{2}\sqrt{\frac{\mu\rho_g}{2u_0\bar{r}}}. \tag{21}$$

According to the results of [1, 2, 9] and others, $\bar{r} = A/u_0$ with the coefficient $A = (2\text{-}3)\cdot 10^{-3}$ m^2/sec. Taking this into account in (21), we obtain $m \ll 0.2$ kg/m^3, i.e., it is sufficient to select $m \approx 0.02$ kg/m^3. According to experimental results [9], drop coagulation is negligible up to $m = 0.05$ kg/m^3.

NOTATION

r, drop radius, m; ρ_f, ρ_g, fluid and gas densities, respectively, kg/m^3; σ, surface tension, N/m; u, velocity of drop blowing, m/sec; u_0, gas velocity in the cylindrical section of the pipeline, m/sec; x, spacing from the entrance to the cylindrical section, m; Q_f, fluid flow rate, kg/sec; Q_g, gas flow rate, m^3/sec; S(x), area of pipeline section, m^2; v(x), gas velocity in the section x, m/sec; v(r, x), drop velocity in the section x, m/sec; m, M, drop masses per unit volume of gas and space, respectively (in a fixed reference system), kg/m^3; g(r, x), $g_s(r, x)$ mass size distributions of drops per unit volume of gas and space, respectively; t, time, sec; μ, gas viscosity, N·sec/m^2; $\mathrm{Re} = 2ru\rho_g/\mu$, Reynolds number for the drops; $I(l)$, light intensity passing through an aerosol layer of thickness l; $f_s(r)$, counting distribution function; n, N, numbers of drops per unit volume of gas and space, respectively; K(r), light-scattering coefficient by a drop; $\bar{r}$, mean Sauter radius of the drops.

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